STATIONARY AND NON-STATIONARY TIME SERIES

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Abstract

The paper „Stationary and Non-stationary Time Series" presents in a theoretical approach, the concept of time series, its characteristics which are: variability, homogeneity, periodicity and interdependence of time series terms, from which result the methods of estimation and analysis of time series, specific for each component (trend, periodical component-cyclical or seasonal, random component). As time series terms have or havn't an evolution in time, it is stationary and non-stationary. In the case of stationary time series in the paper are defined the white-noise processes and the autoregressive processes. Since most processes are non-stationary by transformations we obtain stationary time series and in the paper are presented the restrictions for which an autoregresive process of moving average $ARMA(p,q)$ is stationary. In the paper are approached also non-stationary processes by deterministic type and by stochastic type as the way to achieve stationary which mean the trend estimation and its elimination from the initial time series.

Keywords: stationary time series, non-stationary time series, white-noise process, autoregressive moving average process, autocorrelation function.

JEL Classification: C10

INTRODUCTION

The time series is a set of numerical observations registered in different time moments or intervals such as: annual profits, quarterly production, weekly sales, daily exchange rate a.s.o. The analysis of time series means first the analysis of the nature of variation in these series. The resulted information offer an image concerning with variables behaviour in time and serve to making forecasting of the terms values of time series. The analysis of time series begins with their graphic representation which shows a great variety of movements, some of them with systematic nature such as production growth, others with a more or less systematic variation in connection with a year periods and others with a random occurrence. In the analysis of time series is necessary to measure the systematic movements of data, to identify a certain tendency in their evolution, a certain periodicity which occurs on long term-ciclicity or within a year-seasonality, a certain irregularity which mean to analysis the four components of series which are the trend and cyclical, seasonal and random variations. Among the methods used for the analysis of the components of time series, the method based on the moving averages has a special importance both for statistical and econometrical approach. In this paper we want to identify the characteristics of time series, the already mentioned components and, depending on the existence or non-existence of the trend, the characteristics of stationary and non-stationary time series.

CHARACTERISTICS OF TIME SERIES

The time series consists from two parallel rows of numbers in which the first row shows the variation of time characteristic and the second the change of the analysed variable from a time unit to another. So we define the time series as a succession of real values given by the relation:

$$y_1, y_2, ..., y_T \text{ or } (Y_t)_{t=1,...,T} \tag{1}$$

where y_t represents the level of Y variable at a given moment or for a time period noted with t and T is the number of terms from its background. Each value of series is the achievement of a random variable Y_t. All the random variables $(Y_t)_{t=1,...,T}$ defined a random process. A time series is discrete if $t \in Z$ or is continuous if $t \in R$.

In the estimation and analysis of time series are important their characteristics which are: variability, homogeneity, periodicity and interdependence of time series terms. In the analysis of social and economical time series we must assure the following conditions: the homogeneity of time series, the interdependence within the series, the statistical comparability of series terms.

The time series terms *variability* results from that each term is obtained by the centralization of some individual different data as a development level because the action of some essential causes and also because of an great enough number of non-essential causes whose association usually change from one period to another. So, the estimation and analysis of time series need the measurement of the degree and the form of essential factors influence and the degree of deviation from the general tendency determined by the non-essential, random factors influence.

The time series terms *homogeneity* shows that such kind of series consists from the same type of data which are the result of the same laws action. The analysis of time series needs for each series to verify its terms homogeneity which mean all the time series terms to be expressed in the same unit of measure and for the value indicators to take into account also by the prices evolution. In the case of some long economical and social time series is possible to appear breaks of slope or level as result of some essential modifications of the analysed phenomenon in the considered time interval. In such cases, the average estimation in the all data series isn't relevant because the average is a significant value only if the series terms are qualitatively similar. When in the preliminary analysis there is more qualitatively different subperiods is recommended to divide the series in subseries, to determine first averages on subperiods and subsequently an average for the whole series.

We consider a series with n terms, which is symbolized by (E,n) and which presents $k-1$ breaks of level. So we have the series: $(E_i, p_i); i = 1 \div k$.

We define the following relations:

$$E = \bigcup_{i=1}^{k} E_i ; n = \sum_{i=1}^{k} p_i \qquad (2)$$

The analysis of time series which presents breaks of level needs the following steps: the identification of subseries and the estimation of their synthetic indicators and if is possible, the identification of the periodicity of appearance the breaks of level within the series. So, the time series will be studied concerning with their stability comparing with a certain trajectory (as a rule, shows interest the analysis of series stability comparing with a straight line parallel with the time axis) or with the series trend. A great value of the variation coefficient can result from the existence of breaks of slope and/or level within the series and in this case it is recommended to divide the series in subperiods or it can shows that the data series didn't presents a regular trend in its evolution. In case in which from the graphic representation of the data series results breaks of slope or level, the question is the analysis of the degree of similarity among subperiods. In this purpose for each subperiods $(E_t, p_t), t = 1 \div k$ are estimated: mean, variance, standard deviation and, if it is necessary, variation coefficient. For the whole series (E,n) is calculated, starting from the series terms or from the subperiods indicators, the appropriate averages. In the aim to establish if among subperiods there are substantial differences it can be used more methods such as:

- decomposition of total variance of the series in partial variances (and the estimation of their average) and the variance among subperiods (due to break factors) and the establishing of the weight of variance due to break factors (as much as this is bigger the differences among subperiods are bigger);
- using the analysis of variance by estimation the variation determined by the break factors, variation within the period, total variation and the interpretation of the value of statistics F .

The time series terms *periodicity* results from the choice of time unit at which the data are registered, depending on the content and the measurement possibilities of each indicator. So, some variables are recommended to be measured in small units of time (for example, exchange rate is daily registered, inflation rate, employment monthly a.s.o.) and others in great units of time (for example, turnover, company's net income, macroeconomic indicators of results such as: gross domestic product, net domestic product, net exports a.s.o. are yearly registered). Depending on the nature of influence factors that determine the appearance of oscillations in the evolution of the time series terms and the periods dimension at which is manifested repeatability are known cyclical and seasonal variations. So, some time series highlight situations when macroeconomic variables present oscillations around the trend, usually sinusoidal type, caused by internal factors of economy, by the interrelationships among the different sides of this, repeated at unequal time intervals and bigger than an year, called *cyclical oscillations* (1). The time series which highlights the situations when some macroeconomic variables are influenced by the change of seasons are estimated and analysed as time series with *seasonality* (for example, tourism demand, level of harvest at agricultural crops, construction activity a.s.o. are recorded monthly or quaterly).

Frederick C. Mills has presented in his work "Statistical Methods" two methods of determination the cyclical oscillations which are: residual method and Method of the National Bureau of Economic Research (USA). The application of the *residual method* in the analysis of cyclical oscillations involves the following steps:

- correct choise of long term trend for the analysed series;
- elimination the seasonality;
- determination the trend;
- determination of the deviation from the trend of actual data, being eliminated the seasonal variation;
- determining the "cycles" or percentages of deviations from the trend of the actual data.

Method needs the determination of two indicators which are:

- the percentage of trend $(T\%)$, obtained by the reporting the actual values of the series $(Y_t)_{t=1,...,T}$ to the adjusted values $(\hat{Y}_t)_{t=1,...,T}$, using the formula:

$$T\% = \frac{Y}{\hat{Y}} \cdot 100 \qquad (3)$$

- *residue cyclically in relative expression* $(RC\%)$, which expresses the percentage of deviation from trend of each values of series, computed with the formula:

$$RC\% = \frac{Y - \hat{Y}}{\hat{Y}} \cdot 100 \qquad (4)$$

Graphical representation of percentage of trend gives a suggestive image of cyclical component deviation from trend line. Residual method don't allows a prediction of cyclical variation, but only the analysis of evolution of real series. When monthly or quarterly data are available and graphic representation of the series suggests the presence of seasonal phenomenon, the application of residual method requires the elimination in advance from the actual data of seasonal variations using one of the known methods for determination of monthly (quarterly) seasonality indices and the influencing the empirical values with these indices. In subsequent calculations is used the deseasonalized time series.

Method of the National Bureau of Economic Research (USA) is a more complex method having the aim to answer at two questions which are:

- if in the concrete time series exists a form of variation that repeats (with smaller or larger deviations) in the consecutive economical cycles of the general economic activity (commercial or industrial) and which are the characteristics of this variation. This question concerns the behavior of various time series in the consecutive phases of the enlivening and depression of economic situation taken as a whole.
- if in the concrete time series exists an own undulation movement and which are its characteristics. This question refers to the determination of the cyclical component from the time series and it can be solved using the residual method.

Essentially, the application of this method supposes the achievement of a reference model by its using to be established, in their historical evolution, the lower and upper points of the industrial and commercial activity. Subsequently, the specific cycles overlap the reference cycles, with the aim to establish the compliance or lack of compliance among the phases of general economic cycle and specific courses.

There are three main reasons to study the seasonality which are:

1. determination the seasonality allow to establish the previous development model of the phenomenon and by this we obtain valuable information connecting with the predicted evolution of a phenomenon or process, into a present period.
2. determination the seasonality is useful for the achievement of the forecasting calculation of the seasonal variations, essential for the short-term decisions.
3. once established the existence of the seasonal component we can eliminate its effects from time series, operation called *deseasonalisation*. The determination of the seasonal effects allows therefore or the elimination of these influences and the study of the remained variations or the study separate of the seasonal variations.

The basic method of analysis the seasonality is that after we have established that the seasonal factors are manifested additive or multiplicative, the values of components are estimated for each season of the analysed period, by determination of the seasonal deviations or the seasonal indices.

In the case of non-stationary time series, such as there are the most economical and social time series, the estimate of seasonal deviations needs previously trend determination with an analytical model or with method of least squares and identification subsequent identification of seasonal oscillations.

The time series terms *interdependence* results from the fact that they are successive values of the same variables recorded at certain intervals or moments of time, which make that the value of each term depends by the level from the past. Such a dependence can be surprised by the functional relation:

$$y_t = f(y_{t-1}, \ldots, y_{t-k}), k \geq 1 \qquad (5)$$

where k represent the number of terms from past which have a significant influence on the value y_t.

In the analysis of a time series we must make the distinction between:

- *temporal connection*, which supposes the ordering of a variable values by their temporal succession;
- *causal connection*, which supposes that if an event A occurred after the event B, then the elimination of the event B will determine necessarily the elimination of the event A.

Analysis of time series is based on the simultaneous existence of the temporal and causal connection.

Are defined two events which are:

A - value of variable Y to be y_t at the moment t;

B - value of variable Y to be y_{t-1} at the moment $t-1$.

We consider $P(A)$ and $P(B)$ the probabilities of achievement the events A and B and $P(A/B)$ conditional probability, so that A to be preceded in time by B.

We describe the probabilistic causal connection by the following formula:

$$P(Y = y_t | Y = y_{t-1}) \succ P(Y = y_t) \qquad (6)$$

Interdependence among the time series terms is shown by the *autocorrelation* of time series terms which explains the objective tendency of development the phenomena. Autocorrelation among time series terms creates difficulties in estimating parameters of the adjustment equation, therefore its existence should be identified from the graphic representation of the time series and the establishing of its level by estimating the *autocorrelation coefficient*.

The calculus relation is:

$$\rho_k = \frac{\sum_{t=1}^{n-k}(y_t - \bar{y}_1)(y_{t+k} - \bar{y}_2)}{\sqrt{\left[\sum_{t=1}^{n-k}(y_t - \bar{y}_1)^2\right]\left[\sum_{t=1}^{n-k}(y_{t+k} - \bar{y}_2)^2\right]}} = \frac{\operatorname{cov}(y_t, y_{t+k})}{\sigma(y_t)\sigma(y_{t+k})} \quad (7)$$

where k represents the number of periods in which are separated the pairs of values and $\bar{y}_1$ and $\bar{y}_2$ are calculated with the relations:

$$\bar{y}_1 = \frac{\sum_{t=1}^{n-k} y_t}{n-k} \text{ and } \bar{y}_2 = \frac{\sum_{t=1}^{n-k} x_{t+k}}{n-k} \quad (8)$$

In the case of stationary time series or for n large enough, case in which $\bar{y}_1 = \bar{y}_2$, the autocorrelation coefficient is given by the relation:

$$\rho_k = \frac{\sum_{t=1}^{n-k}(y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{n-k}(y_t - \bar{y})^2} \quad (9)$$

If the values ρ_k are graphic represented depending on k, the graphic so obtained is named *correlogram* and it illustrated the principal characteristics of the autocorrelation phenomenon of the time series terms.

STATIONARY TIME SERIES

Considering the given values of time series (y_t) with $t = 1 \div n$, than:

1. this series defined a stationary process in a restricted sense, if $E(y_t) = constant$ for any t and the correlation matrix of series $y_{1+\tau}, y_{2+\tau}, ..., y_{n+\tau}$ don't depend on τ;
2. this series is named white-noise if it is a stationary process with null average and the autocorrelation coefficients satisfy the property $\rho_k = \begin{cases} 1, if\ k = 0 \\ 0, if\ k \neq 0 \end{cases}$.

Considering the time series given by the relation (1) we introduce the following definitions:

- a process $\{Y_t\}_{t \in Z}$ is *strictly stationary* if for any $(t_1, ..., t_n) \in T^n$, with $t_1 \prec t_2 \prec ... \prec t_n \subset R$ defining the time axis and any $h \in T$ so that $t_i + h \in T$, the processes $\{Y_{t_i+h}\}_{t=1}^n$ and $\{Y_{t_i}\}_{t=1}^n$ have the same distribution;
- a process $\{Y_t\} t \in Z$ is a stationary process by m order if for any $(t_1, ..., t_n) \in T$ with $t_1 \prec t_2 \prec ... \prec t_n$ and any $h \in T$ so that $t_i + h \in T$, for the following two processes, $\{Y_{t_i}\}_{i=1}^n$ and $\{Y_{t_i+h}\}_{i=1}^n$ verify equal averages:

$$E\left((Y_{t_i})^{m_1}, ...(Y_{t_n})^{m_n}\right) = E\left((Y_{t_1+h})^{m_1}, ..., (Y_{t_n+h})^{m_n}\right) \quad (10)$$

where the parameters $m_1, ..., m_n \in N^*$ verify inequality $m_1 + ... + m_n \leq m$.

In the case of *stationarity of first order*, in which $m = 1$ and $m_1 = 1$ obtains equal averages: $E(Y_t) = E(Y_{t+h}) = \mu$. In a stationary process, its characteristics (average, variance a.s.o.) are constant in time.

A stationary process noted $\{Y_t\}_{t \in T}$, where $T = R, N$ or Z have the following properties:

1. $E(Y_t) = m$, $\forall t \in T$;
2. $\operatorname{var}(Y_t) = \sigma_y^2$, $\forall t \in T$;
3. $\operatorname{cov}(Y_t, Y_s) = E(Y_t Y_s) - m^2 = \gamma(|t - s|)$, $\forall (t, s) \in T^2$.

The third property of the stationary process leads at a new definition of the seasonality of a stochastic process which is: a process $\{Y_t\}_{t\in Z}$ is *stationary of the second order* or is slight stationary if this satisfies the following three properties:

1. $E(Y_t)=m$, $\forall t\in T$;
2. $\operatorname{var}(Y_t)=\sigma_y^2$, $\forall t\in T$;
3. $\operatorname{cov}(Y_t,Y_{t+h})=\gamma(h)$, $\forall (t,h)\in T^2$.

In the case of a stationary process of second order the achievements of covariance function are determined by the difference in time and not by the actual values of time series. Stationarity of second order is called also *stationarity in covariance* (2).

In the case of non-stationary series by data transformations we can obtain a stationary time series. If the series is non-stationary and non-homogeneous both the mean and variance are varied over time. In such a kind of series first recourse to the transformation of the series by logarithm, which is a special case of Box-Cox transformation. In the case of series given by the relation $(Y_t)_{t=1\div T}$ this is:

$$T_\lambda(Y_t)=\begin{cases}\dfrac{Y_t^\lambda-1}{\lambda}, \lambda\in(0,1)\\ \log Y_t, \lambda=0\end{cases} \tag{11}$$

Choosing the optimal value of the parameter transformation needs the using of more values of this and choose the one that ensures the lower value of the sum of squares residues or it can use the ***maximum verosimility method.*** Transforming data through logarithm apply if the variation coefficients of the original series are constant in time, so the average and the standard deviation have proportional sizes to time. After logarithm, for the series so obtain apply the proceeding of stationalizing applied to a homogeneous non-stationary series.

For example, the time series $\{Y_t, t\in R\}$, defined by the relation $Y_t=b+at+\varepsilon_t$, where ε_t is a white-noise isn't stationary because the average of process defined by $EY_t=b+at$ isn't independent by t. To obtain a stationary series apply the first order differential operator, resulting the series:

$$V_t=Y_t-Y_{t-1}=a+\varepsilon_t-\varepsilon_{t-1} \tag{12}$$

In the case of transformed series we have the following characteristics:

1. average of transformed series is time independent, because $EV_t=a$;
2. variance is independent of the time variable because $\operatorname{var}(V_t)=\operatorname{var}(a+\varepsilon_t-\varepsilon_{t-1})=2\sigma_\varepsilon^2$;
3. series covariance is time independent, because:

$$\operatorname{cov}(Y_t,Y_{t+h})=E(\varepsilon_t-\varepsilon_{t-1})(\varepsilon_{t+h}-\varepsilon_{t+h-1})=-E(\varepsilon_t\varepsilon_{t+h-1})-E(\varepsilon_{t-1}\varepsilon_{t+h-1})$$

$$=\begin{cases}-\sigma_\varepsilon^2, h\in\{-1,1\}\\ 2\sigma_\varepsilon^2, h=0\\ 0, h\in Z-\{-1,0,1\}\end{cases} \tag{13}$$

Series obtained from this transformation has not the characteristics of white-noise.

So let's consider that (ε_t) is a white-noise process and (σ_y^2) is the variance of this process. When we refer at two or more white-noise process we use symbols such as (ε_{1t}) and (ε_{2t}). So, with these notations we can build a model of time series with the form:

$$x_t=\sum_{i=0}^{q}\beta_i\varepsilon_{t-i} \tag{14}$$

For each period t, x_t construct from the values $\varepsilon_t,\varepsilon_{t-1},...,\varepsilon_{t-q}$ multiplied by the value associated β_i. A sequence formed in this way is named moving average of q order and it is noted $MA(q)$. By combining a moving average process with a linear differential equation we obtain an autoregresive moving average model.

Considering the differential equation of p order with the form:

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + x_t \tag{15}$$

and $\{x_t\}$ a process of moving average of q order, that is $MA(q)$, we can write:

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \sum_{i=0}^{q} \beta_i \varepsilon_{t-i} \tag{16}$$

We follow the convention of normalizing the unit so that β_0 is always equal with the unity. If the given characteristic roots of the relation (16) are all inside an unit circle than $\{y_t\}$ is called autoregresive moving average model $(ARMA)$ for y_t. The autoregresive part of the model is the differential equation, given by homogeneous portion (15) and the part of moving average is the sequence $\{x_t\}$. If the homogeneous differential equation contains p lags and the model for x_t q lags, the model is named $ARMA(p,q)$. If $q = 0$, the process is called pure autoregresive, noted $AR(p)$ and if $p = 0$, the process is a pure moving average process, noted $MA(q)$.

Let's try to identify the necessary and sufficient conditions for an autoregresive process of order 1 $AR(1)$ to be one stationary process. We consider the process:

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t \tag{17}$$

Also we consider that the process started in the period zero, so that y_0 is the initial deterministic initial condition. The solution of such equations is:

$$y_t = a_0 \sum_{i=0}^{t-1} a_1^i + a_1^t y_0 + \sum_{i=0}^{t-1} a_1^i \varepsilon_{t-i} \tag{18}$$

If we can use the term limit value (18), than the sequence $\{y_t\}$ will be stationary. For any given y_0 and $|a_1| \prec 1$, results that t must be enough. Indeed, if a survey is generated by a process which has recently begun, the recorded values can not be stationary and from this reason researchers based on the assumption that the data generation process took place in an infinitely long time. Without the original value y_0, the sum of homogeneous and particular solutions for y_t is:

$$y_t = a_0 / (1 - a_1) + \sum_{i=0}^{\infty} a_1^i \varepsilon_{t-i} + A(a_1)^t \tag{19}$$

where A is an arbitrary constant.

Stability conditions are the following:

1. homogeneous solution must be zero. The sequence must begin at infinity in the past or the process must always be balanced (so that arbitrary constant to be zero);
2. characteristic root a_1 must be less than unity, in absolute value.

Generalizing these tow conditions for all the processes $ARMA(p,q)$, results that a necessary condition that these processes to be stationary is the homogeneous solution to be zero.

Also, is demonstrated that for any moving average process MA, given by the relation (16), necessary and sufficient conditions for it to be stationary are that: $(1)\ \sum(\beta_i)^2$ and $(2)\ (\beta_s + \beta_1\beta_{s+1} + \beta_2\beta_{s+2} + \ldots)$ to be zero (3).

Considering an autoregressive moving average process, given by the relation (15), with x_t given by the relation (14), if roots of homogeneous form (15) are inside of an unit circle and the sequence $\{x_t\}$ is stationary, than the process $\{y_t\}$ is stationary, this being the condition of stationarity of the autoregresive coefficients.

NON-STATIONARY AND NON-HOMOGENEOUS TIME SERIES

Most of economic data series are not stationary, they havn't constant average and display phases of relative stability followed by periods of high variability. Many of present econometrical research are based on the extended Box-Jenkins methodology in the behavior analysis of these type series. Usually, the using of autoregressive methods needs these series stationalization.

Also the most macroeconomic variables are non-stationary. The sample averages are not constant and/or there is o strong tendency of *heteroskedasticity*. We can identify same key-characteristics of the great variety of time series from this domain which are:

1. *many time series of macroeconomic variables contain a clear trend* (for example, gross domestic product and its components, profit rates, inflation rates a.s.o. have positive trends which may be followed by periods of decline and therefore it is difficult for such variables have a constant mean, so they are not stationary);
2. *some series seem to oscillate* (for example, exchange rates don't show a clear tendency of increase or decrease: in some periods currency is assessing, in othres are depreciating and this random motion is specified non-stationary series);
3. *any impact registered by a series has a highly degree of persistence*;
4. *many time series variability isn't constant in time.* Such kind of series are *conditionally heteroskedastic* if the unconditional variance (or on long-term) is constant, but there are periods in which variance is relatively high.
5. *some series are similar in development with other*s. Such simultaneous movements should not surprise but we must understand that the forces which influence the national economy have a similar influence on the international economy.

Analysis of a time series identifies two types of non-stationary series which are:

1. *processes of deterministic non-stationary type - TS (Trend Stationarity)*;
2. *processes of non-stationary stochastic type - DS (Difference Stationarity).*

Each of these two processes have certain characteristics and trend removal and conversion into stationary processes require different techniques.

A non-stationary process by deterministic type is reprezented by the relation:

$$y_t = P_t + \varepsilon_t \tag{20}$$

where P_t is a polynomial of a certain degree in relation to variable time and ε_t is a stationary process.

If the polynomial is first-degree, case in which $P_t = \alpha + \beta t$ and ε_t is a white-noise, then the process is characterized by the relation:

$$y_t = \alpha + \beta t + \varepsilon_t \tag{21}$$

Such a process has the following characteristics:

1. the process average is dependened by the time, dependency characterized by the relation: $E(y_t) = \alpha + \beta t$;
2. the process variance is constant in time, this being equal with the variance of white-noise which is: $\text{var}(y_t) = \text{var}(\alpha + \beta t + \varepsilon_t) = \sigma_\varepsilon^2$;
3. covariance of this process is zero: $\text{cov}(y_t, y_{t'}) = 0$.

In the case of such a process the tendency has a deterministic nature and the variance is constant over time. Passing this deterministic non-stationary process type by a stationary process is done by estimating the trend and elimination it from the original series. Therefore, the stationalization is achieved in two phases namely:

1. are estimated polynomial parameters P_t by using the method of least squares. If the trend has obtained a linear model: $\hat{y}_t = \hat{\alpha} + \hat{\beta} t$.
2. stationary series is obtained by removing the estimated trend in the first stage, namely: $\hat{\varepsilon} = y_t - \hat{y}_t = y_t - (\hat{\alpha} - \hat{\beta} t)$.

Such a stationary series can be represented by an autoregressive process of a particular type.

A *non-stationary stochastic process* has the representation: $y_t = \rho y_{t-1} + \beta + \varepsilon_t$, where ε_t is a stationary process and ρ, β are real constant.

When $\rho = 1$ non-stationary stochastic process allows the representation: $\Delta y_t = \beta + \varepsilon_t$.

As we shown most economic series are not stationary, having the following characteristics of non-stationarity: the series has an average which is not constant over time, this following usually a linear path with positive or negative slope (series is non-stationary homogeneous type), such series characterizing by constant changes from one period to another; there are series which have variation non-constant from one period to another, case in which the series is non-stationary by non-homogeneous type, both variance and average being variables over time. For establishing whether the time series is stationary are applied different statistical tests between the methods most commonly used being the analysis of autocorrelation function (4). If the series respects one of the following criteria, it is non-stationary and must be stationalized by differentiation:

1. autocorrelation function values are in the vicinity of one or are negative;
2. a number of successive values of autocorrelation function are relatively equal.

When one of the two cases is done, than the series $\{y_t\}_{t=1 \div T}$ is differentiated and we obtain $\Delta y_t = y_t - y_{t-1}$, $t = 2,...,T$. For the new series is resumed the analysis of stationality on the base of estimation the autocorrelation function.

Among the criteria of choosing the most adecquate autocorelation model we include: *classical criteria*, by which is chosen that model which has the highest value for R^2 adjusted or the lowest variance or dispersion residues; *indicators based on information theory*, providing the autoregressive model selection for which the criterion Akaike or Schwartz has the lowest value.

The formal identification of the trend presence in time series is realized by statistical tasting, as we saw or by graphic representations.

CONCLUSIONS

We see from this brief overview of the characteristics of time series the importance of identifying them, analytical or graphical, their components and then the presence or not the trend. *Variability* in time series results from the fact that on the analysed phenomenon act essential causes which determines the general tendency of the time series terms and non-essential random causes, which determines the degree of deviation from trend. Variability has determined the application of some tests for identification the type series (stationary or non-stationary), frequently used to determine the stationarity being the analysis of autocorrelation function. If the series is non-stationary, the stationalization is the first step in selecting the most appropriate model for the trend. For stationary or stationarized time series are identified the parameters of autoregressive models, is chosen the most appropriate model that can be used later in analysis and forecasting. In this paper we tried to establish conditions for stationarity of a time series in an autoregressive moving average process.

ENDNOTES

(1) *Cyclical oscillations are determined by the periodicity of succession of different economic processes such as the periodic renewal of the production unit of society, the formation and the cyclical reproduction of primary resources (fixed capital, power resources and raw materials, labor), the introduction in the exploitation of some new resources, the development in leaps of science and technological innovation, the periodic restructuring of national economy, the social revolutions, wars a.s.o. (source: Biji, E. M., Lilea, E., Roşca, R. E., Vătui, M., Statistică aplicată în economie, Edited by Universal Dalsi, Bucharest, 2000, p., 362).*

(2) *An example of stationary series is the white-noise ε_t, for which are valid the three properties: 1. $E(\varepsilon_t) = 0$, $\forall t \in T$; 2. $\mathrm{var}(\varepsilon_t) = \sigma_\varepsilon^2$, $\forall t \in T$; 3. $\mathrm{cov}(\varepsilon_t, \varepsilon_h) = 0$, $\forall t \neq h$. (source: Andrei, T., Bourbonnais, R., Econometrie, Edited by Editura Economica, Bucharest, 2008, p. 341)*

(3) *If the condition* (2) *is valid for all values of* s *and* $\beta_0 = 1$*, than the condition* (1) *is redundant and a moving average process of finite order* MA *will be always stationary. For a process of finite order, the condition (2) is true for all values* $s \geq 0$*, where* s *is the number of periods. (source: Enders, W., Applied econometric time series, 1st edition, Edited by John Wiley & Sons, Inc., USA, 1995, p. 76-77).*

(4) *Stationalization of time series is the first stage in using the Box-Jenkins procedeure for selecting the most appropriate model of trend. Briefly, the steps of this procedure are: calculate indicators for stationalized analysis (when the time series is non-stationary it will be stationalized by proper transformations, for example, logarithm the series values or apply a Box-Cox transformation, than differentiate the data series thus obtained). If it is a stationary series is identifying the type of model, the parameters of model are estimated using an autoregressive known method such as: the method of least squares, the maximum likelihood method, the method of moments a.s.o., are tested the characteristics of estimated autoregresive models, are choosed the best model, are achieved various analysis and forecasts. (source: Andrei, T., Bourbonnais, R., Econometrie, Edited by Editura Economică, Bucharest, 2008, p. 368).*

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