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JOB DISCRIMINATION AGAINST WOMEN AND ENDOGENOUS POPULATION CHANGE IN A GENERALIZED SOLOW GROWTH MODEL

Prof. **Wei-Bin ZHANG**
Ritsumeikan Asia Pacific University, Japan
wbz1@apu.ac.jp

Abstract:
This study examines economic growth and population change with discrimination against women in the labor market within the analytical framework of Solow's neoclassical growth model. The study models dynamic interactions between wealth accumulation, time distribution between work, children caring, and leisure, population change with endogenous birth and mortality rates with gender discrimination. The production technology and markets are built on Solow's neoclassical growth model. The basic mechanisms for population changes in the Barro-Becker fertility choice model and the Haavelmo population model are integrated to model the population change. This study also takes account of discrimination against woman in the labor market. We synthesize these dynamic forces in a compact framework by applying Zhang's utility function. The model properties are studied by simulation. We find equilibrium points and illustrate motion of the dynamic system. We also examine the effects of changes in the discrimination against woman, the propensity to save, woman's propensity to pursue leisure activities, the propensity to have children, woman's human capital and man's emotional involvement in children caring.

Key words: gender discrimination; birth and mortality rates; propensity to have children; gender difference in time distribution; population growth; wealth accumulation

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1. INTRODUCTION

Since the publication of *An Essay on the Principle of Population* in 1798, economists proposed different ideas about interdependence between population change and economic growth. In the last two hundred years, economies in different parts of the world have experienced different patterns of population changes. These changes include, for instance, rapid aging, raising life expectancies, and falling fertility rates in industrialized economies. This study deals with dynamic interactions between wealth accumulation and population dynamics with birth rate, mortality rate and gender discrimination against women. The unique contribution of this study is through introducing gender discrimination in labor market we demonstrate the role of the discrimination in time distribution and population dynamics.

This paper introduces population growth with endogenous birth and mortality rates into Solow's neoclassical one sector growth model with gender discrimination. The economic production and markets are the same as modelled in neoclassical growth theory. The seminal paper in the field is the Solow model (Solow, 1956; and Burmeister and Dobell, 1970). Neoclassical growth theory studies endogenous physical capital or wealth accumulation in perfectly competitive markets (see, for instance, Azariadis, 1993; and Barro and Sala-i-Martin, 1995). Our model is constructed on the basis of neoclassical growth theory. We model economic production and wealth accumulation within the framework of the Solow model. However, household behavior is modelled by applying Zhang's (Zhang, 1993).

Population change rate is the net result of birth and mortality rates. One finds many factors in the literature of population dynamics which are supposed to be related to birth rates (Barro and Becker, 1989; Galor and Weil, 1996; Adsera, 2005; Chu *et al.*, 2012; Hock and Weil, 2012). According to Becker *et al*. (1990), it is very costly to bring up children to adulthood and to provide them education. There are quality-quantity trade-offs on children. For instance, Galor and Weil (1999) and Doepke

(2004) argue that the transition of economies from a stage of stagnation to perpetual growth may be strongly affected quality-quantity trade-offs on children. In a study by Bosi and Seegmuller (2012) heterogeneity of households is examined in terms of capital endowments, mortality, and costs per surviving child. Varvarigos and Zakaria (2013) study interdependence between fertility choice and expenditures on health. Their model is influenced by the studies on fertility and health expenditures by Bhattacharya and Qiao (2007) and Manuelli and Seshadri (2009). According to Varvarigos and Zakaria predicts one may find a fall in fertility in association with the process of economic growth. There are many other models on dynamics of mortality rates (e.g., Kirk, 1996; Ehrlich and Lui 1997; Acemoglu and Johnson, 2007; Strulik, 2008; Galor, 2012). Aging is another important topic in modern economics. For a given population structure, aging population and mortality rate are closely related. It is thus significant to understand social and economic mechanisms of mortality (Cigno and Rosati, 1996; Robinson and Srinivasan, 1997; Schultz, 1993, 1998; Blackburn and Cipriani, 2002; Chakraborty, 2004; Hazan and Zoabi, 2006; Heijdra and Romp, 2008; Ludwig and Vogel, 2009; Lee and Mason, 2010; Balestra and Dottori, 2012; Lancia and Prarolo, 2012; and Ludwig *et al.*, 2012). Zhang (2014) builds a dynamic model of population change under influences of these studies. This study bases Zhang's model in describing the population dynamics.

To explain birth and mortality rates we need to take account of gender differences in behaviour. As Flabbi (2010: 745) argues: "Even if wages and earnings for women and men in the United States have experienced a significant convergence in the 1970s and 1980s, their ratio has remained roughly constant at 75% since the mid-1990s… The United States is not an exception among OECD countries: they rank more or less average, with Northern European countries traditionally showing the lowest differentials and Japan the highest. These differentials persist after conditioning on observable productivity characteristics… ." This study is to address issues related to how discrimination against women may affect population. Becker (1957) points out that in association with more intensive competition in production, one might expect that costly discrimination will become weaker. According to Black and Brainerd (2004: 541), "The recent narrowing of the gender earnings gap in an era of increased competition through international trade and deregulation might seem to offer support for this theory. Since 1960, the time trend for the female: male wage ratio has closely tracked that for imports as a share of GDP, with both series remaining fairly constant between 1960 and 1980, then increasing dramatically through the early 1990s". Endogenous preferences should help to explain complicated patterns of gender division of time (e.g., Goodfriend and McDermott, 1995; Kelly, 1997; Antecol, 2000; Edmonds and Pavcnik, 2006). This study incorporates gender discrimination issues, gender differences in time distributions. The model is to integrate Zhang's two models. Zhang (2014) develops a model of economic growth with a constant population and gender division of labor and discrimination against women in labor market. This paper introduces into this model. Zhang (2015) develops a growth model with endogenous birth and mortality rates and endogenous population. This study proposes a model to reveal interdependence between population growth and discrimination against women by synthesizing the two models in a compact framework. The paper is organized as follows. In Section 2 we develop the basic model with endogenous wealth and population. In Section 3 we simulate the model. In Section 4 we make comparative dynamic analysis with regard to some parameters. In Section 5 we conclude the study.

2. THE BASIC MODEL

We follow the Solow growth model in describing the production sector (Solow, 1956). Markets are perfectly competitive. The economy has a single production sector. There is a single commodity which is used for consumption and investment. Capital depreciates at a constant exponential rate δ_k. which is independent of the manner of use. Technology of production sector is constant returns to scale. Factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is made by households. Assets of the economy are owned by households. All their incomes are spent on consumption, saving, and child bearing. The population is composed of male and female populations. Each gender's adult and young populations are homogeneous. A family is composed of

consists of husband, wife and children. All the families are identical. Subscript indexes $q=1$ and $q=2$ are used to represent man and woman respectively. The variable $N(t)$ represents the adult population of each gender. We use $T_q(t)$ and $\bar{T}_q(t)$ to represent work time and time spent on taking care of children of gender q. Let $\bar{N}(t)$ represent the total labor supply employed in time t for production. We use $N_q(t)$ to represent the qualified labor force of gender q. We have

$$N_q(t) = h_q T_q(t) N(t), \quad \bar{N}(t) = N_1(t) + N_1(t), \tag{1}$$

where h_q is gender $q's$ level of human capital.

The production sector

The production sector employs two inputs - capital and labor. We use $K(t)$ and $F(t)$ to represent capital stock and output level. The production function specified as

$$F(t) = AK^{\alpha}(t)\bar{N}^{\beta}(t), \quad \alpha, \beta > 0, \quad \alpha + \beta = 1, \tag{2}$$

Where α and β are respectively the constant output elasticities of capital and qualified labor input and A is the total factor productivity. Labor and capital are paid their marginal products. Firms earn zero profits. Let $w(t)$ stand for the wage rate of per unit of qualified work time in fair labor market where workers earn their marginal value of labor. Nevertheless, there is gender discrimination in labor market (e.g., Heyman, et al. 2013; Jonathan and Kerwin, 2013; Lanning, 2014). We describe discrimination on the basis of Zhang (2014). There is a fraction φ of women's fair share of the gender's labor taken away by firms from women. The rate φ is called the discrimination rate against woman. As observed by Dozier *et al*. (2013: 13): "Gender discrimination cannot be measured by directly asking respondents in a survey if they systematically discriminate against women with regard to salaries. Since such conduct is illegal, that question would yield only normative responses. Thus, we are left with the somewhat unsatisfactory methodology of testing any variables that might mediate the relationship between gender and income. We treat the residual variance as a quantified estimate of gender discrimination." As demonstrated later on, by comparative dynamic analysis we can get some insights into possible effects of the discrimination rate.

The total cost of the female labor force due to discrimination is $\varphi h_2 T_2(t) N(t)$. The production sector's profit is given as follows

$$F(t) - (r(t) + \delta_k)K(t) - w(t)h_1 T_1(t)N(t) - (1-\varphi)w(t)h_2 T_2(t)N(t),$$

The marginal conditions imply

$$r(t) + \delta_k = \frac{\alpha F(t)}{K(t)}, \quad w_1(t) = \frac{\beta h_1 F(t)}{\bar{N}(t)}, \quad w_2(t) = \frac{\beta h_2 F(t)}{\bar{N}(t)}, \tag{3}$$

where $w_q(t)$ is the wage rate of per unit of work time by gender q

$$w_1(t) \equiv w(t)h_1, \quad w_2(t) \equiv (1-\varphi)w(t)h_2.$$

The current and disposable incomes

We apply Zhang's approach to modelling consumer behaviour (Zhang, 1993). There are five variables for the representative household to decide: consumption level of commodity, leisure time, work time, number of children, and saving. Variable $\bar{k}(t)$ stands for wealth per household, i.e., $\bar{k}(t) = K(t)/N(t)$. We have the representative household's current income $y(t)$ from the interest and wage payments as follows

$$y(t) = r(t)\bar{k}(t) + w_1(t)T_1(t) + w_2(t)T_2(t).$$

The representative household can also sue the value of $\bar{k}(t)$. Selling and buying wealth are assumed to conducted instantaneously without any transaction cost. This assumption is evidently strict, but enables to solve households' decision problem. The representative household's disposable income is the sum of the current income and the value of wealth

$$\hat{y}(t) = y(t) + \bar{k}(t). \quad (4)$$

The cost of children caring

We use $n(t)$ and $p_b(t)$ to represent the birth rate and the cost of birth at time. Many factors are found to be related to costs of bringing up children. As in Zhang (2014), it is assumed that children have the same level of wealth as that of parent. Parents spend time and the following cost on their children

$$p_b(t) = n(t)\bar{k}(t). \quad (5)$$

It should be remarked that Barro and Becker (1989) include consumption of goods as a part of the cost. Becker (1981) takes account of costs of the mother's time spent on rearing children to adulthood. This study proposes a relation between fertility rate and the parent's time on raising children as follows

$$\bar{T}_q(t) = \theta_q n(t), \ \ \theta_q \geq 0. \quad (6)$$

The relation implies that the more the parents want children, the more they spend time on child caring. We do not possible changeable return to scales.

The budget and time constraint

The total available budget is between saving, $s(t)$, consumption of goods, $c(t)$, and bearing children, $p_b(t)$. We express the budget constraint as follows

$$p(t)c(t) + s(t) + \bar{k}(t)n(t) = \hat{y}(t). \quad (7)$$

In addition to work and child caring, parents have their own leisure time. We denote the leisure time of gender q by $\tilde{T}_q(t)$. T_0 is used to stand for the available time for work, children caring leisure, and leisure. Each worker has time constraint as follows

$$T_q(t) + \bar{T}_q(t) + \tilde{T}_q(t) = T_0. \quad (8)$$

Insert (8) in (7)

$$p(t)c(t) + s(t) + \bar{k}(t)n(t) + \bar{T}_1(t)w_1(t) + \bar{T}_2(t)w_2(t) + \tilde{T}_1(t)w_1(t) + \tilde{T}_2(t)w_2(t) = \bar{y}(t), \quad (9)$$

where $\bar{y}(t) \equiv (1 + r(t))\bar{k}(t) + (w_1(t) + w_2(t))T_0$.

We consider $\bar{y}(t)$ as "potential" income which the family can have if they spend all the available time on work. The left-hand side of (9) means the sum of the consumption cost, opportunity cost of leisure, saving, and opportunity cost of bearing children. Insert (6) in (9)

$$c(t) + s(t) + \tilde{w}(t)n(t) + \tilde{T}_1(t)w_1(t) + \tilde{T}_2(t)w_2(t) = \bar{y}(t), \quad (10)$$

where

$$\tilde{w}(t) \equiv \bar{k}(t) + h\,w(t), \ \ h \equiv \theta_1 h_1 + (1-\varphi)\theta_2 h_2.$$

The opportunity cost of children fostering is given by $\tilde{w}(t)$.

The utility and optimal behavior

We follow Barro and Becker (1989) in that that the parents' utility is assumed to be dependent on the number of children. Following Zhang (2015), the utility is a function of $c(t)$, $s(t)$, $\tilde{T}_q(t)$, and $n(t)$

$$U(t) = c^{\xi_0}(t)s^{\lambda_0}(t)\tilde{T}_1^{\sigma_{01}}(t)\tilde{T}_2^{\sigma_{02}}(t)n^{\upsilon_0}(t),$$

where ξ_0 is the propensity to consume, σ_{0q} the gender $q's$ propensity to use leisure time, λ_0 the propensity to have wealth, and υ_0 the propensity to have children. Maximizing the utility subject to the budget constraint yields

$$c(t) = \xi\,\bar{y}(t), \ \ s(t) = \lambda\,\bar{y}(t), \ \ \tilde{T}_q(t) = \frac{\sigma_q\,\bar{y}(t)}{w_q(t)}, \ \ n(t) = \frac{\upsilon\,\bar{y}(t)}{\tilde{w}(t)}, \quad (11)$$

where

$$\xi \equiv \rho\xi_0,\ \ \lambda \equiv \rho\lambda_0,\ \ \sigma_q \equiv \rho\sigma_{q0},\ \ \upsilon \equiv \rho\upsilon_0,\ \ \rho \equiv \frac{1}{\xi_0 + \lambda_0 + \sigma_{10} + \sigma_{20} + \upsilon_0}.$$

Population change with endogenous birth and mortality rates

We use $n(t)$ and $d(t)$ to stand for the birth rate and mortality rate, respectively. The population change rate is birth rate minus mortality rate

$$\dot{N}(t) = (n(t) - d(t))N(t), \tag{12}$$

The birth rate is determined by (11). On the basis of different approaches in the literature of economic growth with endogenous population (e.g., Haavelmo, 1954; Razin and Ben-Zion, 1975; Stutzer, 1980; Yip and Zhang, 1997; Chu *et al.*, 2012), Zhang applies the following equation

$$d(t) = \frac{\overline{\upsilon}\, N^b(t)}{\bar{y}^a(t)}, \tag{13}$$

where $\overline{\upsilon} \geq 0,\ a \geq 0$. We call $\overline{\upsilon}$ the mortality rate parameter. Equation (13) implies that mortality rate is negatively related to the disposable income. As in the Haavelmo model, people live longer in association with improvements in living conditions. In (13) $N^b(t)$ means possible influences of the population on mortality. There are different possible influences. For instance, if environment deteriorates and the population is overpopulated, more people will cause higher mortality. This implies that b is positive. The sign of b is generally ambiguous as the population may have a positive or negative effect on mortality. Insert (10) and (13) in (12)

$$\dot{N}(t) = \left(\frac{\upsilon\,\bar{y}(t)}{\tilde{w}(t)} - \frac{\overline{\upsilon}\, N^b(t)}{\bar{y}^a(t)}\right)N(t). \tag{14}$$

Wealth dynamics

The change in the household's wealth is saving minus dissaving. We thus have

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t) = \lambda\bar{y}(t) - \bar{k}(t). \tag{15}$$

Demand and supply of goods

Output of the production sector equals for the net savings and the depreciation of capital stock. We have

$$S(t) + C(t) - K(t) + \delta_k K(t) = F(t), \tag{16}$$

where $S(t) - K(t) + \delta_k K(t)$ is the sum of the net saving and depreciation and

$$S(t) \equiv s(t)N(t),\ \ C(t) \equiv c(t)N(t),\ \ K(t) \equiv \bar{k}(t)N(t).$$

We built the model with gender discrimination. The model is general as some well-known models, such as the Solow model and the Haavelmo model, can be treated as its special cases.

3. ANALYZING DYNAMIC PROPERTIES OF THE MODEL

This section examines dynamics of the model. We define a new variable $z(t) \equiv (r(t) + \delta_k)/w(t)$.

Lemma

We have the motion of the economy with the following two differential equations

$$\dot{z}(t) = \widetilde{\Omega}_z(z(t),\ N(t)),$$
$$\dot{N}(t) = \widetilde{\Omega}_N(z(t),\ N(t)), \tag{17}$$

in which $\tilde{\Omega}_z$ and $\tilde{\Omega}_N$ are functions of $z(t)$ and $N(t)$ given in the Appendix. All the other variables are given as functions of $z(t)$ and $N(t)$ at any point in time as follows: $\bar{k}(t)$ by (A11) $\rightarrow$ $r(t)$ and $w_q(t)$ by (A2) $\rightarrow$ $\bar{N}(t)$ by (A16) $\rightarrow$ $\bar{y}(t)$ by (A3) $\rightarrow$ $c(t)$, $s(t)$, $\tilde{T}_q(t)$, and $n(t)$ by (11) $\rightarrow$ $\bar{T}_q(t)$ by (6) $\rightarrow$ $T_q(t)$ by (A4) $\rightarrow$ $K(t)$ by (A1) $\rightarrow$ $F(t)$ by (2).

We have two variables $z(t)$ and $N(t)$ and two differential equations. The Appendix shows that the expressions of the two differential equations are very complicated. We simulate the model to plot the motion of the economy. First, we specify $\delta_k = 0.05$ and let $T_0 = 24$. We specify the other parameters as follows

$$\alpha = 0.34,\ \lambda_0 = 0.6,\ \xi_0 = 0.2,\ \nu_0 = 0.4,\ \sigma_{10} = 0.15,\ \sigma_{10} = 0.15,\ A = 1,\ a = 0.4,\ b = 0.5,$$
$$h_1 = 3,\ h_2 = 2.6,\ \theta_1 = 2,\ \theta_2 = 5,\ \bar{\upsilon} = 1,\ \varphi = 0.2. \tag{18}$$

The discrimination rate is 0.2. We fix the propensity to save 0.6 and the propensity to consume 0.3. The total productivity factor is $A = 1$. We assume that the father spends less hours in children fostering than the mother. The parents have the equal propensity to enjoy leisure. The male population has higher human capital than the female population. It should be noted that in the literature of empirical studies on growth, the value of the parameter, α, in the Cobb-Douglas production is often fixed near 0.3 (see, for instance, Miles and Scott, 2005; and Abel *et al*, 2007). To plot the motion of the economy, we choose the following initial conditions

$$z(0) = 1.6,\ N(0) = 21.$$

Figure 1 shows the simulation result. The population falls from its high initial value. Both the birth rate and mortality rate become lower. The wealth rises initially and falls in the long term. The labor force is reduced. The wage rates are enhanced and rate of interest is reduced. The wealth per household and opportunity cost of children fostering are increased. The parents' leisure hours are increased. The falling in birth rate is associated with falling in the parents' time of children fostering. As the income rises, the parents work less hours. The national wealth and output are reduced in association with falling capital and labor force. The consumption level and the representative household's wealth rise.

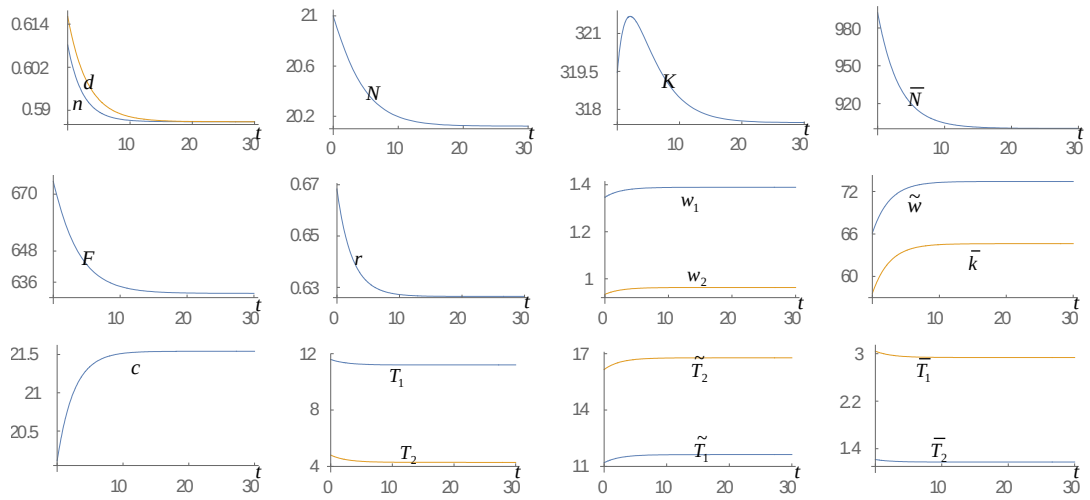

Figure 1. The Motion of the Economic System

We can show that the system becomes stationary in the long run. Simulation confirms that the system has an equilibrium point. The equilibrium values of the variables are listed as follows

$$N = 23.73,\ K = 446.49,\ \bar{N} = 1260.3,\ F = 885.6,\ n = d = 0.59,\ r = 0.62,$$
$$w_1 = 1.86,\ w_2 = 1.11,\ \tilde{w} = 89,\ \bar{k} = 78.4,\ T_1 = 11.6,\ T_2 = 2.3,\ \tilde{T}_1 = 12.3,\ \tilde{T}_2 = 18.8,$$

$\overline{T}_1 = 1.17$, $\overline{T}_2 = 2.93$, $c = 26.1$.

The system's two eigenvalues at the equilibrium point are: -0.393 and -0.369. As the two eigenvalues are negative, the unique equilibrium is locally stable. Hence, the system always approaches its equilibrium point if it is not far from the equilibrium point.

4. COMPARATIVE DYNAMIC ANALYSIS

The motion of the national economy was plotted under (18). It is significant to carry out comparative dynamic analysis as parameters are changeable and changes in a parameter has transitory and long-run impact. The Lemma in the previous section gives the computational procedure to calibrate the movement of the economy. This implies that we can study the transitory and long-run effects of change in any parameter. We introduce a variable $\overline{\Delta}x(t)$ to represent the change rate of the variable $x(t)$ in percentage due to changes in the parameter value.

4.1. The discrimination rate against woman rises

As mentioned in Zhang (2014), there are different ways of discrimination against women. Boserup (1970) holds that economic growth and the status of women should exhibit a curvilinear relationship. According to Boserup, there is a widening gap between men and women in initial stages of economic growth. In their empirical research on rural Bangladesh's patterns of women's work and determinants of the gender division of labor in, Bose *et al*. (2009) find that the gender division of labor is determined both by economic and socio-cultural factors. Nevertheless, few formal models take accord of gender discrimination in the literature of economic development. We now increase the discrimination rate against women in the following way: $\varphi: 0.2 \Rightarrow 0.25$. Man's wage rate is increased and woman's wage rate is reduced. The opportunity cost of children caring is reduced. As a consequence of strengthened discrimination, the wife works less hours has more leisure hours and the husband works more hours and less leisure hours. The wife and husband increase children fostering time initially and reduce fostering time in the long term. Both birth and mortality rates are increased initially and are reduced in the long term. The population is reduced. The qualified labor force rises initially and falls in the long term. Both consumption and wealth levels are reduced. The rate of interest falls. The output rises initially and falls in the long term.

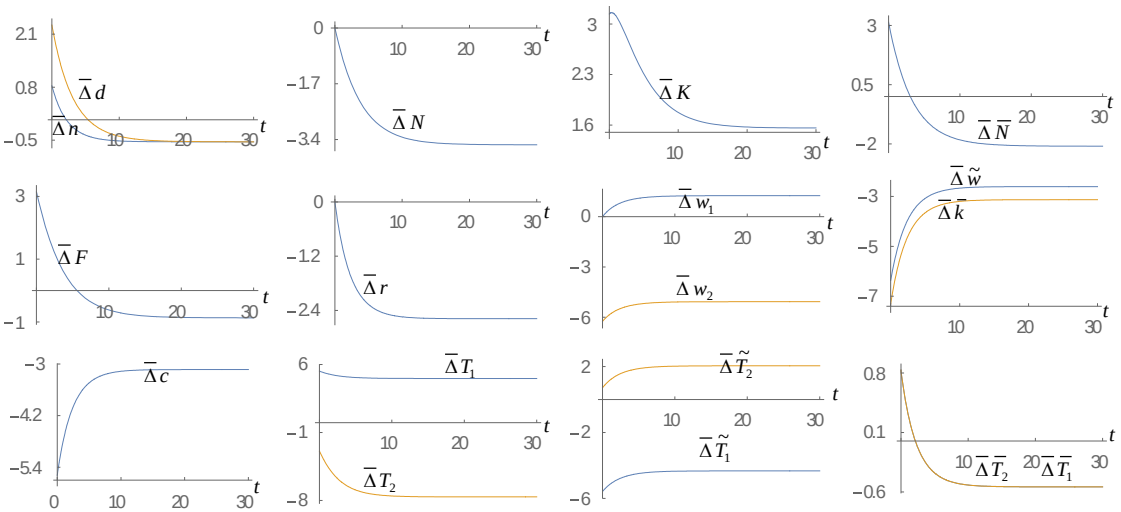

Figure 2. Stronger Discrimination against Women

4.2. Woman's human capital being improved

According to the traditional neoclassical approach gender inequalities due to disparities in human capital will wither away when an economy has high economic growth (e.g., Beneria and Feldman, 1992; Truong, 1997; Forsythe, et al. 2000; Dolado, *et al*. 2001; Duflo, 2012). Stotsky (2006: 18) holds "the neoclassical approach examines the simultaneous interaction of economic development and the reduction of gender inequalities. It sees the process of economic development leading to the reduction of these inequalities and also inequalities hindering economic development." Although this study assumes human capital exogenous, we can fully describe a change in human capital on the economic system. We now enhance the mother's human capital as follows: $h_2 : 2.6 \Rightarrow 2.8$. The results are plotted in Figure 3. As the mother accumulates more human capital, her wage income is increased. As the mother earns more per unit time, she works more and has less leisure time. The opportunity cost of child fostering is increased as the mother's wage rises. The father's wage is slightly affected. The father works less and stays longer at home. Both the mother and father shorten time of children fostering. The family consumes more goods and owns more wealth. The parameter change enhances the capital, total labor input and output. The mortality rate is reduced as living conditions are improved. The net impact of falling birth and mortality rates augments the population. It should be noted that some researches find positive interdependence between life expectancy and the aggregate human capital level (e.g., Blackburn and Cipriani, 2002; Boucekkine *et al*., 2002). Our result also demonstrates the same trend if we consider the mortality rate negatively related to the life expectancy. The positive population growth has relatively weak impact on the birth rate. If woman's human capital and preference for leisure are increased simultaneously, the birth rate may fall faster than the mortality rate. If this happens, then the population will fall.

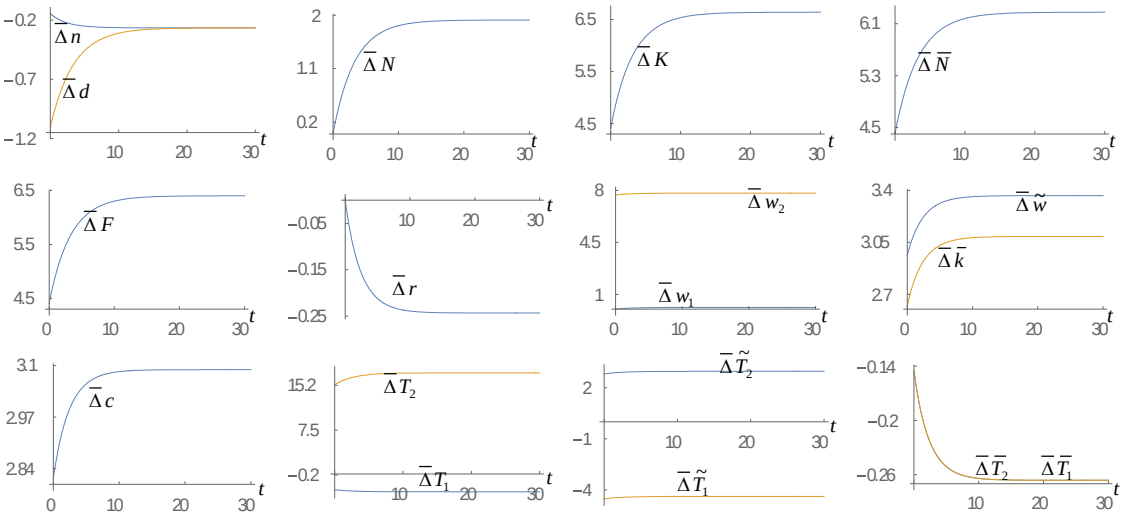

Figure 3. A Rise in Woman's Human Capital Being Improved

4.3. A rise in the propensity to save

The Solow model predicts that a rise the propensity to save brings about an increase in per capita wealth but lowers per capita consumption level. The population growth rate is not affected by economic conditions in the traditional one-sector neoclassical growth model. We now study effects of saving propensity on population dynamics. We allow the propensity to save to be changed as follows: $\lambda_0 : 0.6 \Rightarrow 0.63$. The results are plotted in Figure 4. The family's wealth is increased in association with the rise in the propensity to save. This increases the opportunity cost of children caring. The wage rates are increased. The rise in the cost reduces the birth rate. Less children means less time spent on children caring. Woman and man have work less hours. Woman and man have less leisure time initially and more in the long term. Both the mother and father spend less hours in children caring. The

population falls as a consequence of falling in the birth and mortality rates. The output and total labor input are reduced in the long term. The national wealth rises.

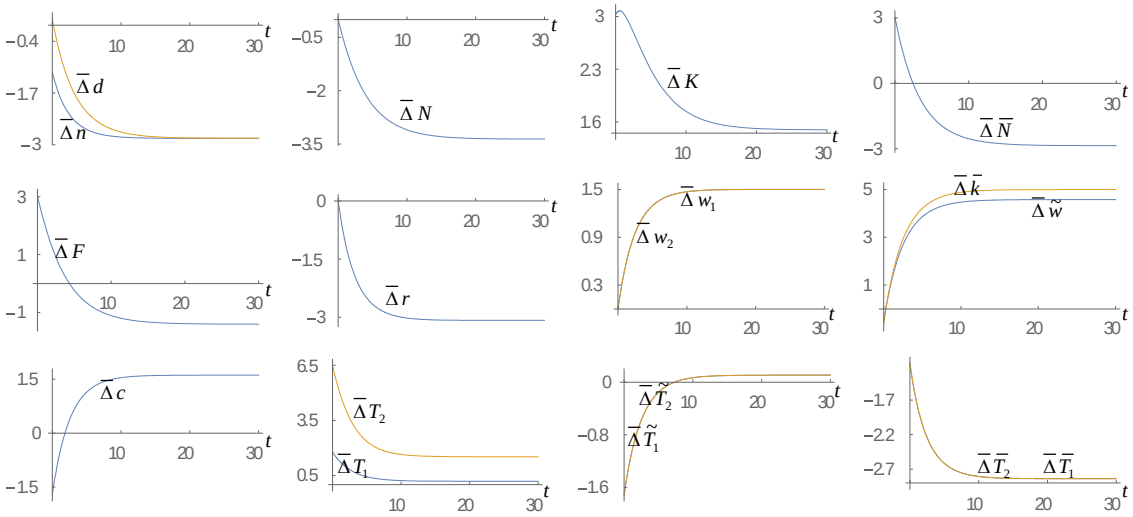

Figure 4. A Rise in the Propensity to Save

4.4. Woman's propensity to enjoy leisure rises

Woman may have different preferences in different stages of economic development. We now study the following rise in women's propensity to enjoy leisure, $\sigma_{02}: 0.15 \Rightarrow 0.16$. The rise in women's preference results in that women stay more hours at home and work less hours. Men have less leisure hours and work more hours. The parents have less hours for children fostering. The wage rates are increased. The consumption level, wealth, and opportunity cost of children fostering fall. The birth rate is reduced. The mortality rate is augmented initially but is reduced in the long term. The population, total labor, wealth and output fall.

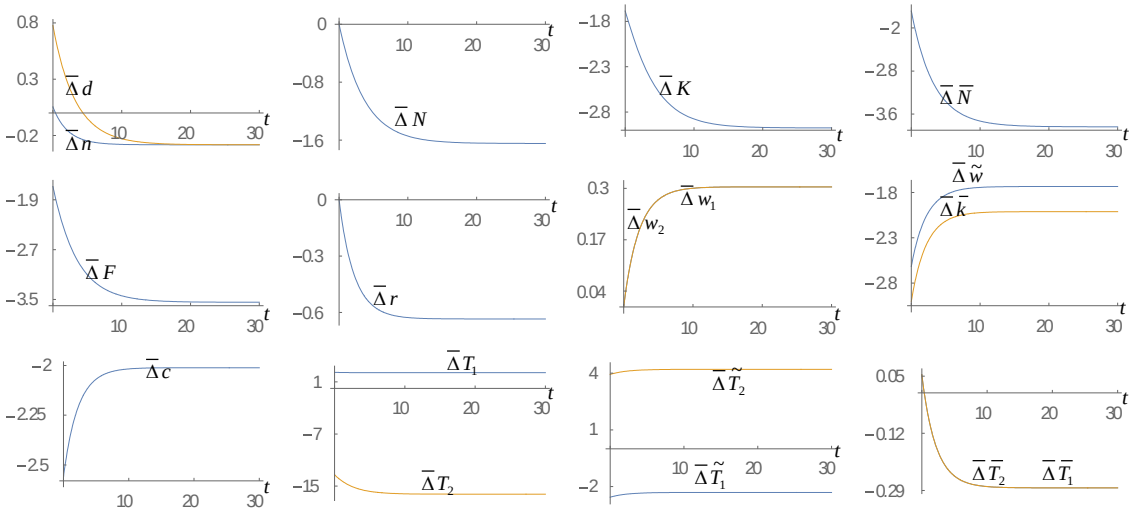

Figure 5. Woman's Propensity to Enjoy Leisure Rises

4.5. An increase in the propensity to have children

The traditional neoclassical growth theory holds that as a national economy is characterized of constant returns to scale, a rise in the population tends to have no impact on per household's living conditions, even though the values of the aggregated variables are affected. Although our model is developed within the neoclassical framework, this study makes the population endogenous. We study the case that the propensity to have children is increased as follows: $\upsilon_0: 0.4 \Rightarrow 0.42$. The simulation

results are plotted in Figure 6. The birth rate and the population rise. The increased population leads to a fall in the mortality initially. In the long term the birth and mortality rates rise. A rise in the propensity to have children have a great impact on the population growth. The capital, total labor input and output level are all reduced initially and enhanced in the long term. As the family has more children, the parents spend more time on children caring. They initially increase their leisure time as they reduce labor hours. Nevertheless, as they wage rates are reduced in the long term, the falling rates in labor time are lowered. More children imply less consumption in the long term. As the change rate of capital is lower than the output, the rate of interest is increased over time. The opportunity cost of children $\tilde{w}$ and the representative household's wealth fall in the long term.

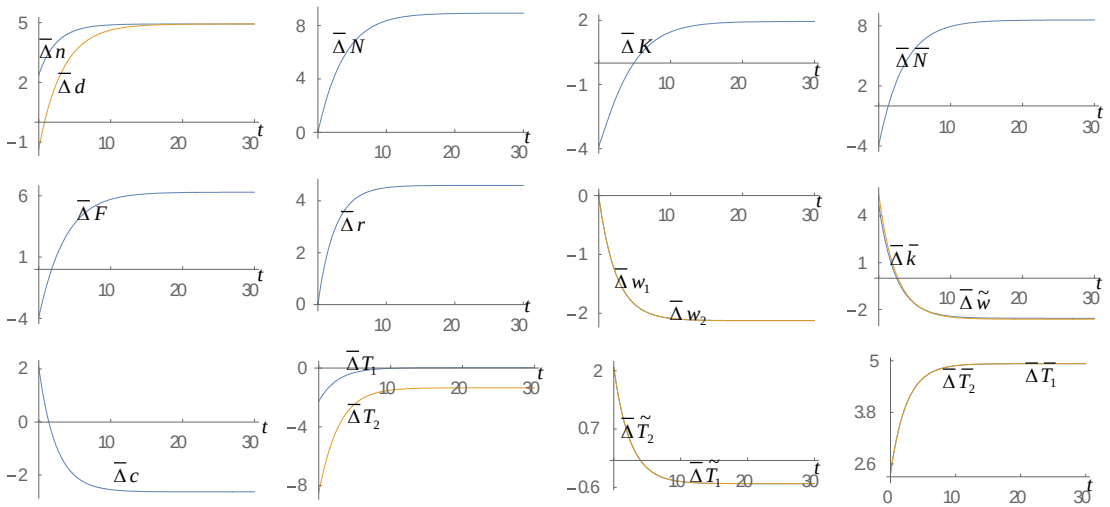

Figure 6. An Increase in the Propensity to Have Children

4.6. The father spending more time on each child fostering

We now consider that the father wants to spend more time with each of his children. We increase the parameter as follows: $\theta_1 : 2 \Rightarrow 2.2$. The father's time on children caring is increased and the mother's time is slightly reduced. The parents spend less hours on leisure. The mother works more hours and the father works less hours. The wage incomes are enhanced for the father and the mother. The opportunity cost of children fostering is increased in the long term. The wealth per household is reduced. The population and the mortality rate fall. The rate of interest falls. The total wealth, total labor input and output fall. The family consumes less.

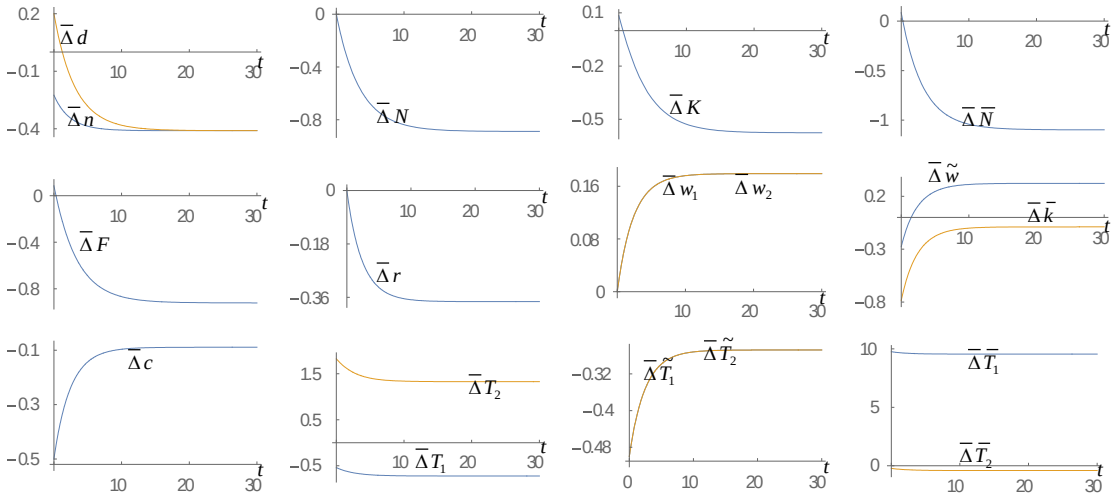

Figure 7. The Father Spending More Time on Each Child Fostering

4.7. A rise in the total factor productivity

The total factor productivity rises as follows: $A:1\Rightarrow 1.05$. The change in productivity enhances the output level and wage rates. The wealth per household and opportunity cost fall initially and increase in the long term. The birth rate is increased and the mortality rate is reduced in the short term. In the long term the birth and mortality rates slightly change. The long-run time distributions change slightly. The national wealth, the population, the labor input, and national output rise.

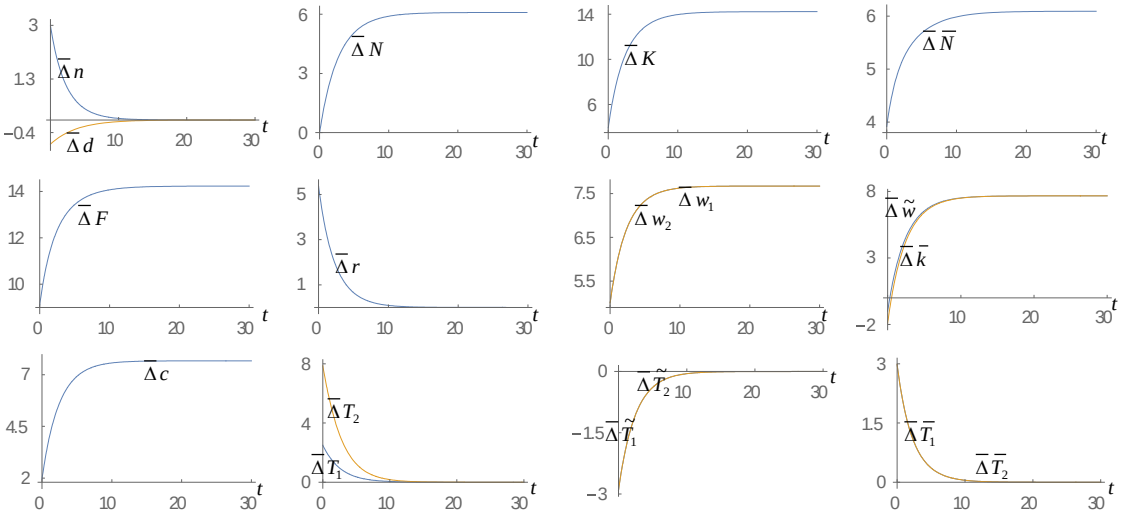

Figure 8. A Rise in the Total Factor Productivity

5. CONCLUDING REMARKS

This study is concerned with economic growth and population change with discrimination against women in the labor market within the analytical framework of the Solow neoclassical growth model. The study built a dynamic model of interdependence between population change with endogenous birth and mortality rates, wealth accumulation, and time distribution between work, leisure and children caring. We built the model on the basis of the Solow growth model, the Haavelmo growth model with endogenous population and the Barro-Becker growth model with endogenous fertility choice. This study also takes account of discrimination against woman in the labor market. We synthesized these determinants of population growth in a compact framework by applying Zhang's utility function proposed by Zhang. The model is simulated for identifying existence of equilibrium points and for plotting motion of the system. We also examined the effects of changes in the discrimination against woman, the propensity to save, the propensity to have children, woman's propensity to pursue leisure activities, woman's human capital and man's emotional involvement in children caring. For instance, when the discrimination rate against women is increased, we have the following changes in the dynamic process: man's wage rate is increased and woman's wage rate is reduced; the opportunity cost of children caring is reduced; the wife works less hours has more leisure hours and the husband works more hours and less leisure hours; the wife and husband increase children fostering time initially and reduce fostering time in the long term; both birth and mortality rates are increased initially and are reduced in the long term; the population and consumption and wealth levels are reduced; the qualified labor force rises initially and falls in the long term; the rate of interest falls; and the output rises initially and falls in the long term. There are many ways to generalize and extend our model. An obvious limitation of our model is that children caring function exhibits constant return to scale in the parent's time spent on children caring. It is possible to generalize our model by using more general utility or production functions. Our research may also be extended and generalized to study some observed phenomena related to gender, human capital and economic development.

APPENDIX: CHECKING THE LEMMA

We show to present the dynamics with two differential equations. From equation (3), we obtain

$$z \equiv \frac{r+\delta_k}{w} = \frac{\tilde{\alpha}\,\bar{N}}{K}, \tag{A1}$$

where $\tilde{\alpha} \equiv \alpha/\beta$. Inserting (A1) in (2) and (3) yields

$$r = \alpha A\left(\frac{z}{\tilde{\alpha}}\right)^{\beta} - \delta_k, \quad w = \beta A\left(\frac{\tilde{\alpha}}{z}\right)^{\alpha}, \quad w_1 = w h_1, \quad w_2 = (1-\varphi) w h_2. \tag{A2}$$

We treat r, w and w_q as functions of z. The definition of $\bar{y}$ and (3) imply

$$\bar{y} = (1+r)\bar{k} + h_0 w, \tag{A3}$$

where $h_0 \equiv (h_1 + (1-\varphi)h_2)T_0$. Use (8) and (11)

$$T_q = T_0 - \bar{T}_q - \tilde{T}_q = T_0 - \left(\frac{\theta_q \upsilon}{\tilde{w}} + \frac{\sigma_q}{w_q}\right)\bar{y}. \tag{A4}$$

Insert (A3) in (A4)

$$T_q = \chi_q - \frac{\tilde{r}_q \bar{k} + \bar{r}_q}{\tilde{w}} - r_q \bar{k}, \tag{A5}$$

where

$$\chi_q = T_0 - \frac{h_0 w \sigma_q}{w_q}, \quad \tilde{r}_q \equiv \theta_q \upsilon (1+r), \quad \bar{r}_q \equiv h_0\, \theta_q \upsilon w, \quad r_q \equiv \frac{(1+r)\sigma_q}{w_q}.$$

Insert (A5) in (1)

$$\frac{\bar{N}}{N} = h_1 T_1 + h_2 T_2 = \chi - \frac{\tilde{r}\,\bar{k} + \bar{h}_0}{\tilde{w}} - \tilde{r}_0 \bar{k}, \tag{A6}$$

where

$$\chi \equiv h_1 \chi_1 + h_2 \chi_2, \quad \tilde{r} \equiv h_1 \tilde{r}_1 + h_2 \tilde{r}_2, \quad \bar{h}_0 \equiv h_1 \bar{r}_1 + h_2 \bar{r}_2, \quad \tilde{r}_0 \equiv h_1 r_1 + h_2 r_2.$$

From (16) we have

$$\bar{\lambda}\,\bar{y} - \delta \bar{k} = \frac{F}{N}, \tag{A7}$$

where $\bar{\lambda} \equiv \lambda + \xi$ and $\delta \equiv 1 - \delta_k$. Insert (A3) and (3) in (A7)

$$(\bar{\lambda} + \bar{\lambda} r - \delta)\bar{k} + \bar{\lambda} h_0 w = \frac{w \bar{N}}{N \beta}. \tag{A8}$$

Insert (A6) in (A8)

$$\left(\frac{(\bar{\lambda} + \bar{\lambda} r - \delta)\beta}{w} + \tilde{r}_0\right)\bar{k} + \frac{\tilde{r}\,\bar{k} + \bar{h}_0}{\tilde{w}} + \beta \bar{\lambda} h_0 - \chi = 0. \tag{A9}$$

From $\tilde{w} = \bar{k} + hw$ and (A9), we have

$$\bar{k}^2 + \tilde{m}_1 \bar{k} + \tilde{m}_2 = 0, \tag{A10}$$

where

$$\tilde{m}_1(z) \equiv \frac{(\bar{\lambda} + \bar{\lambda} r - \delta)h\beta + \tilde{r}_0 h w + \beta \bar{\lambda} h_0 - \chi + \tilde{r}}{\tilde{m}}, \quad \tilde{m}_2(z) \equiv \frac{\bar{h}_0 + (\beta \bar{\lambda} h_0 - \chi)hw}{\tilde{m}},$$

$$\tilde{m}(z) \equiv \frac{(\bar{\lambda} + \bar{\lambda} r - \delta)\beta}{w} + \tilde{r}_0.$$

We solve (A10), treating $\bar{k}$ as the variable

$$\bar{k}(z) = \frac{-\tilde{m}_1 \pm \sqrt{\tilde{m}_1^2 - 4\tilde{m}_2}}{2}. \tag{A11}$$

From (A11) $\bar{k}$ as determined a function of z. We use the following procedure to determine the variables as functions of z and N: $\bar{k}$ by (A11) $\rightarrow$ r and w_q by (A2) $\rightarrow$ $\overline{N}$ by (A16) $\rightarrow$ $\bar{y}$ by (A3) $\rightarrow$ c, s, $\widetilde{T}_q$, and n by (11) $\rightarrow$ $\overline{T}_q$ by (6) $\rightarrow$ T_q by (A4) $\rightarrow$ K by (A1) $\rightarrow$ F by (2). From this procedure and (14), we represent the motion of the population as a function of $z(t)$ and $N(t)$ at any point in time

$$\dot{N}(t) = \widetilde{\Omega}_N(z, N). \tag{A12}$$

We already knew that $\bar{k}$ and $\bar{y}$ are functions of z. Equation (15) implies

$$\dot{\bar{k}} = \Omega_0(z) \equiv \lambda \bar{y} - \bar{k}. \tag{A13}$$

Insert $\bar{k}(t)$ from (A11) in (A13)

$$\dot{z} = \widetilde{\Omega}_z(z) \equiv \Omega_0(z)\left(\frac{d\bar{k}}{dz}\right)^{-1}. \tag{A14}$$

We confirmed the lemma.

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